Parametrization of Generalized Parton Distributions

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No Time for Outline

Main Concepts I would like to get through:

1) Both Exclusive and Semi-Inclusive processes give more information on the deep inelastic structure of hadrons (Ji, Radyushkin, Brodsky, Hwang, Schmidt + factorization studies).

2) However, "what type of information" and "the way to access it" have still to be defined.

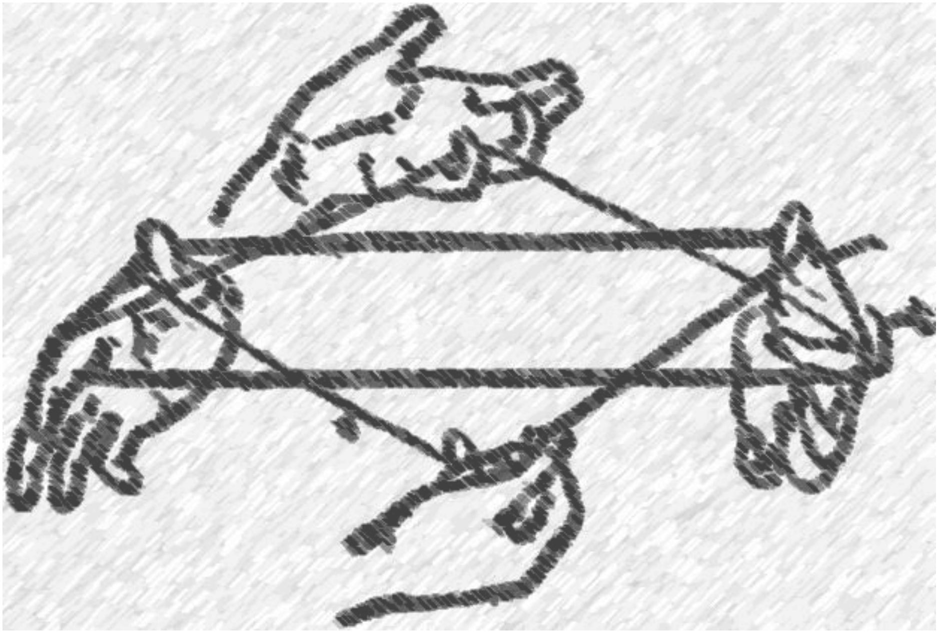
3) These two points pose important theoretical problems. We illustrate a few.

Osvaldo Gonzalez Hernandez, Kunal Kathuria
Gary Goldstein, Swadhin Taneja

Conceptual Issues

No damn cat, and no damn cradle..

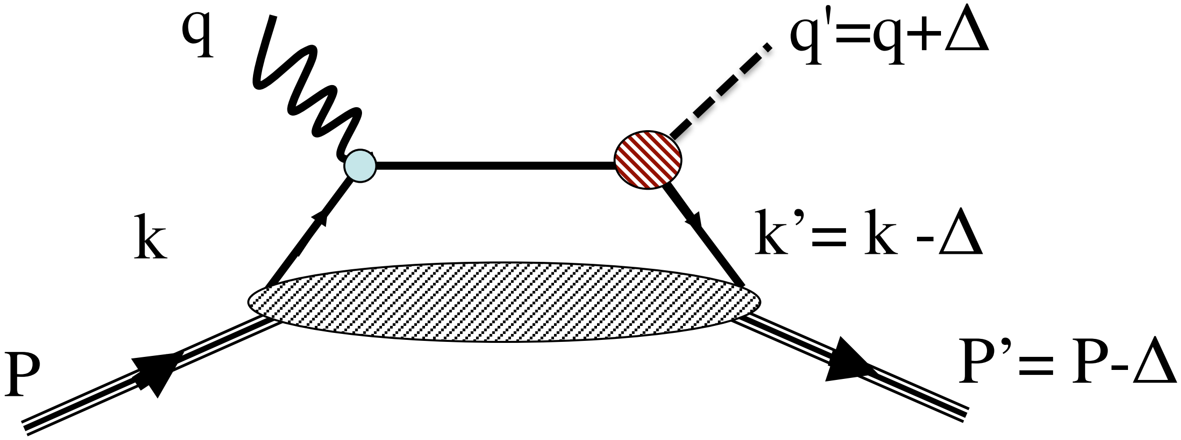
K. Vonnegut
“Cat’s Cradle”

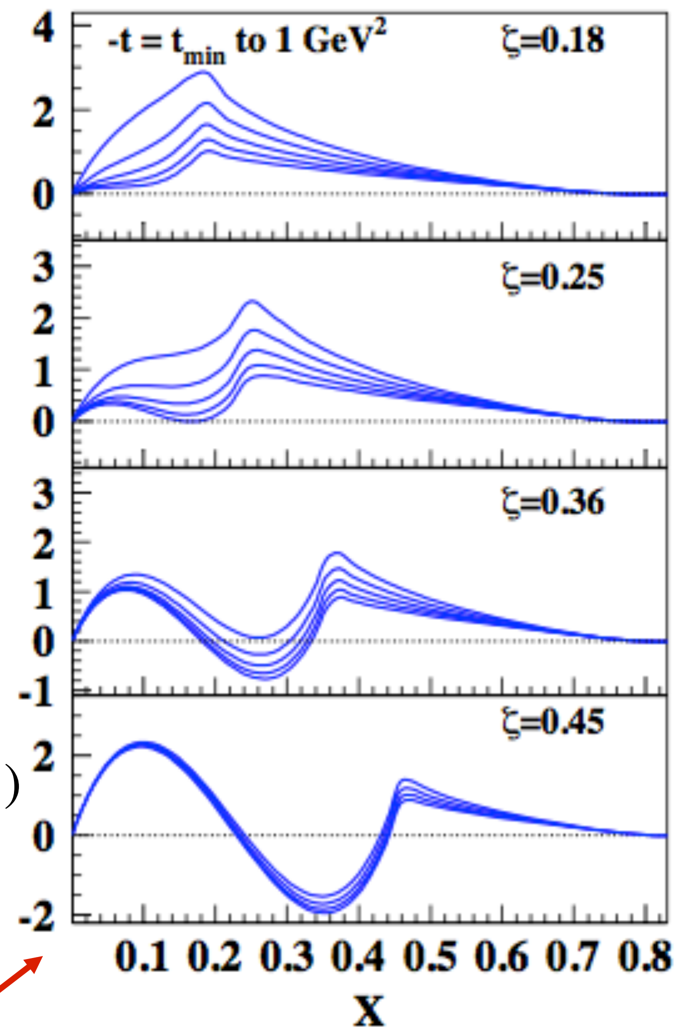


dulcis in fundo….

Off forward Parton Distributions (GPDs) are embedded in soft matrix elements for deeply virtual exclusive experiments

Goldstein et al. arXiv:1012.3776





$$\frac{1}{(p+q)^2 - m^2 + i\varepsilon} = PV\frac{1}{(p+q)^2 - m^2} - i\pi\delta((p+q)^2 - m^2)$$

$$\rightarrow \frac{1}{-Q^2 + 2(pq) + i\varepsilon} \rightarrow \frac{1}{-Q^2/2(Pq) + (pq)/(Pq)} = \frac{1}{-\zeta + X}$$

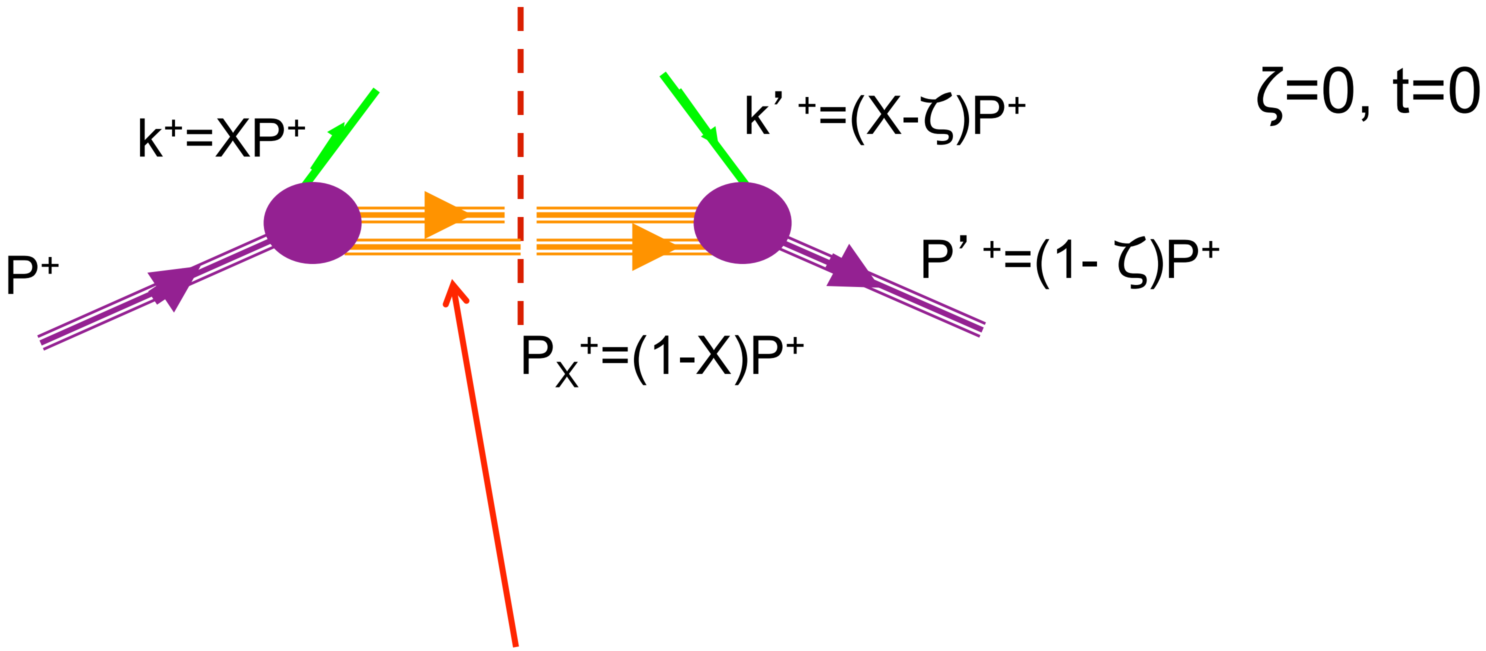
Amplitude

$$\mathcal{F}_q = P.V.\int_{-1+\zeta} dX\, F_q(X,\zeta,t)\left[\frac{1}{\zeta - X} - \frac{1}{X}\right] + i\,\pi e_q^2 F_q(\zeta,\zeta,t)$$

DIS: ζ=0, t=0

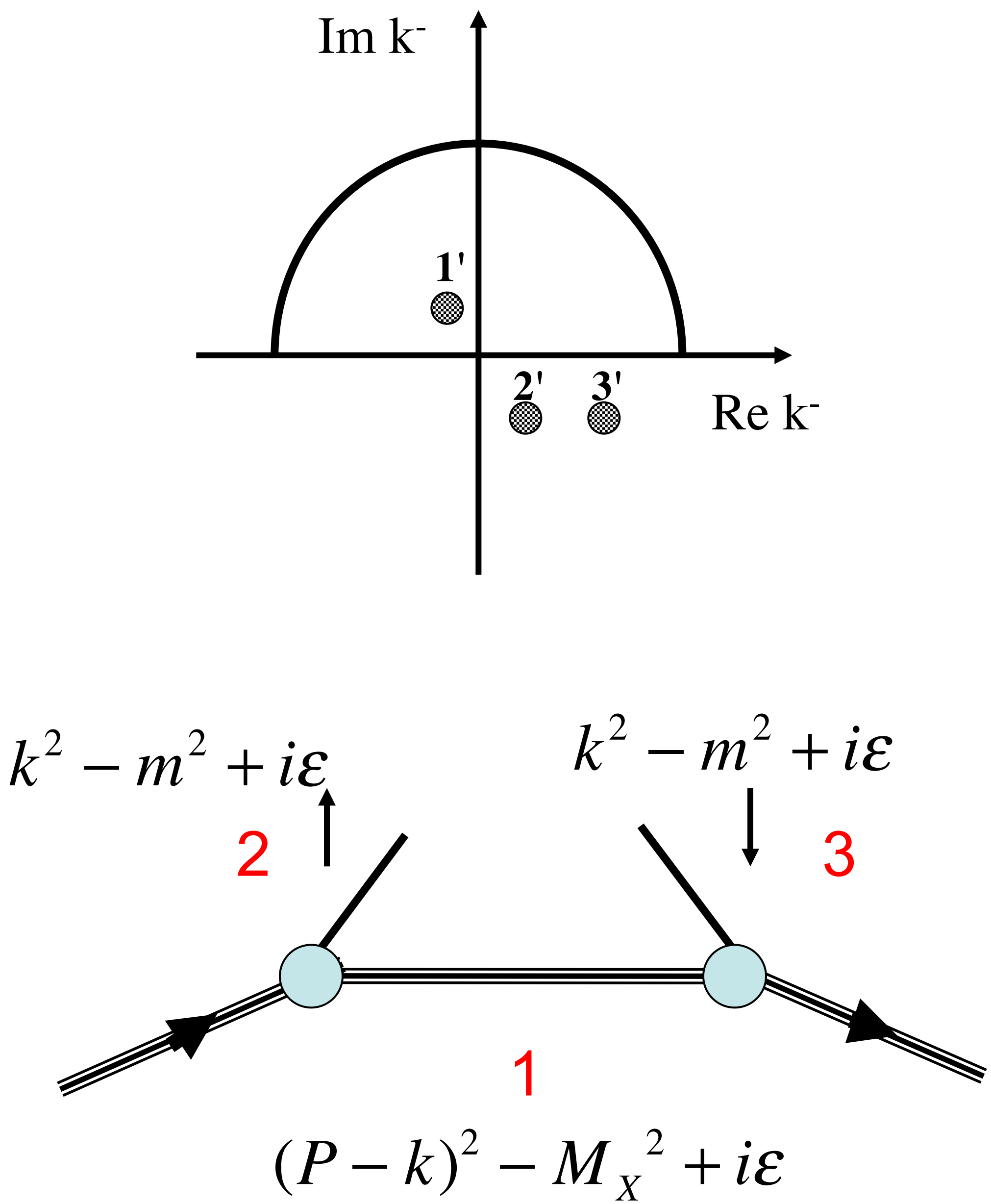
From Amplitude to Cross Section using Optical Theorem

$$\mathcal{F}_q = \mathrm{Re}\,\mathcal{F}_q + i\,\mathrm{Im}\,\mathcal{F}_q \rightarrow \boxed{\sigma_{TOT} \approx \mathrm{Im}\,\mathcal{F}_q(0)}$$



$$H(X,\zeta,t) = \sum_n \langle P'|\psi^+ \boxed{|n\rangle\langle n|} \psi|P\rangle\, \delta\left[(X-\zeta)P^+ + p_n^+ - P'^+\right]$$

From Dispersion Relations (DRs) to OPE



DR

$$\frac{\nu}{M}T(x,Q^2) = 2x\int_{-1}^{+1} dx'\, H(x')\left[\frac{1}{x-x'-i\varepsilon} - \frac{1}{x+x'-i\varepsilon}\right]$$

$$\Downarrow$$

Can be seen as analytic continuation of

$$\frac{\nu}{M}T(x,Q^2) = 2x\int_{-1}^{+1} dx'\, H(x')\left[\frac{1}{x-x'} - \frac{1}{x+x'}\right]$$

$$\Downarrow$$

Taylor expansion

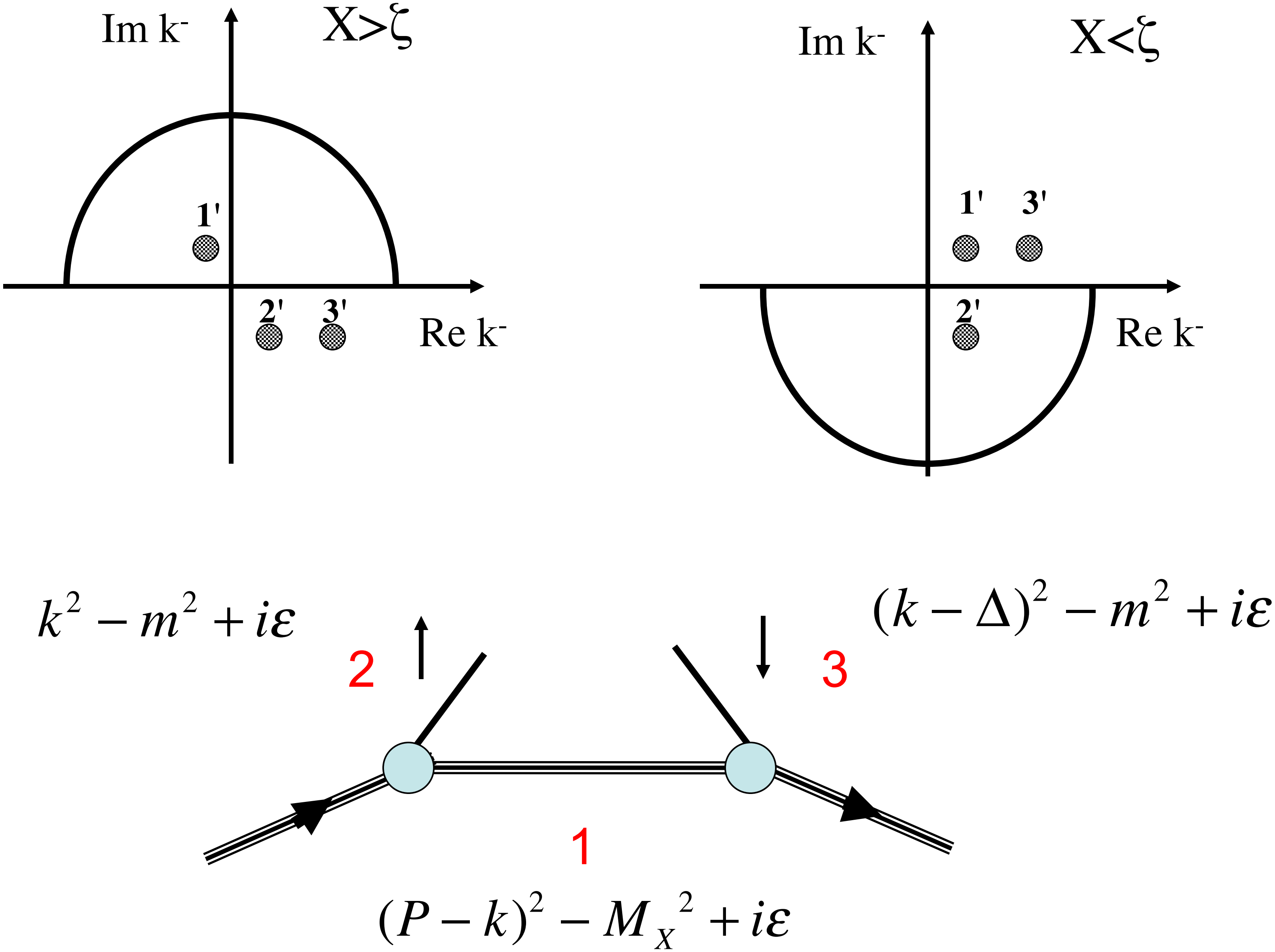
$$\frac{\nu}{M}T(x,Q^2) = 2x\int_{-1}^{+1} dx'\, H(x')\sum_n \left(\frac{x}{x'}\right)^{n-1}$$

$$\Downarrow$$

Mellin Moments ➔ operator product expansion

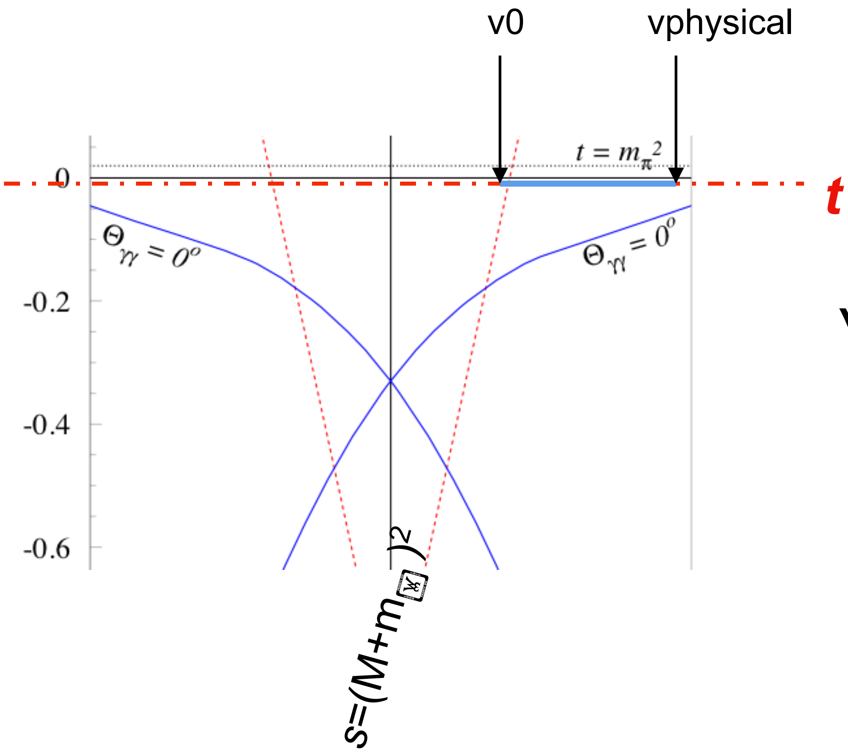
OPE

$$\frac{\nu}{M}T(x,Q^2) = 2x\sum_n \left(\frac{-1}{x}\right)^n M_n \Rightarrow \operatorname{Im} T(x,Q^2) = x\left[H(x) + H(-x)\right]$$


Im k⁻
X>ζ
1'
2'
3'
Re k⁻
Im k⁻
X<ζ
1'
3'
2'
Re k⁻
$k^2 - m^2 + i\varepsilon$
2
3
$(k-\Delta)^2 - m^2 + i\varepsilon$
1
$(P-k)^2 - M_X{}^2 + i\varepsilon$

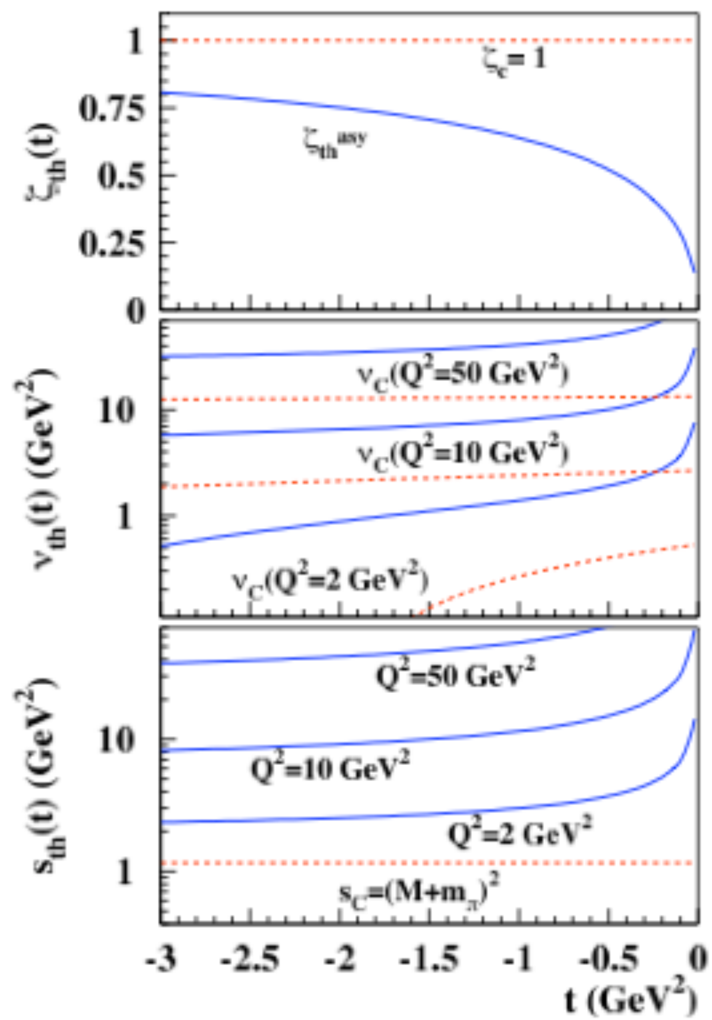
Where is threshold?

The quark + spectator system cannot be on its mass shell but hadronic jets must have some threshold. This threshold ("physical threshold") is much higher than what required for the dispersion relations to be valid



$\nu_{phys} > \nu_o$ if masses in the two-body scattering problem are different!

- Continuum starts at $s = (M+m_\pi)^2$ lowest hadronic threshold.
- How to fill the gap? Analytic continuation?



ζc= 1
ζth^asy
ζth(t)
1
0.75
0.5
0.25
0
νC(Q²=50 GeV²)
νC(Q²=10 GeV²)
νC(Q²=2 GeV²)
νth(t) (GeV²)
10
1
Q²=50 GeV²
Q²=10 GeV²
Q²=2 GeV²
sC=(M+mπ)²
sth(t) (GeV²)
10
1
-3
-2.5
-2
-1.5
-1
-0.5
0
t (GeV²)

x_{Bj}

ν

s

Dispersion relations cannot be directly applied to DVCS because one misses a fundamental hypothesis: "all intermediate states need to be summed over"

This happens because "t" is not zero ➔ t-dependent threshold cuts out physical states

It is not an issue in DIS (see your favorite textbook) because of optical theorem: there is a difference between x and ζ

x is not an observable ➔ its domain is defined in [-1,1]

DRs involve observables ➔ one puts x on the ridge ($x=\zeta$) ➔ observables involve physical thresholds

1) No direct connection between OPE and DRs in DVCS

2) OPE is not affected by physical thresholds but:

t-dependent thresholds are important for experiment: counter-intuitively as Q^2 increases the DRs start failing because the physical threshold is farther away from the continuum one (from factorization)

Is the mismatch between the limits obtained from factorization and the physical limits from DRs a signature of the "limits of standard kinematical approximations"?

When deeply virtual processes involve directly final states - like in exclusive or semi-inclusive processes - "standard kinematic approximations should be questioned"

(Collins, Rogers, Stasto, 2007)

(we write ζ but it is equivalent in ξ), so that

$$H(X,\zeta,t) \to \int dm_J^2 \rho(m_J^2) H(\zeta, \left(1+\frac{m_J^2}{Q^2}\right), \zeta, t)$$

$$H(\xi', \xi', t)/(\xi - \xi')$$

is not the same as

$$\Im m A(\nu', t)/(\nu - \nu')$$

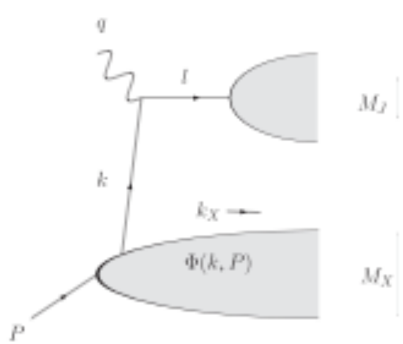
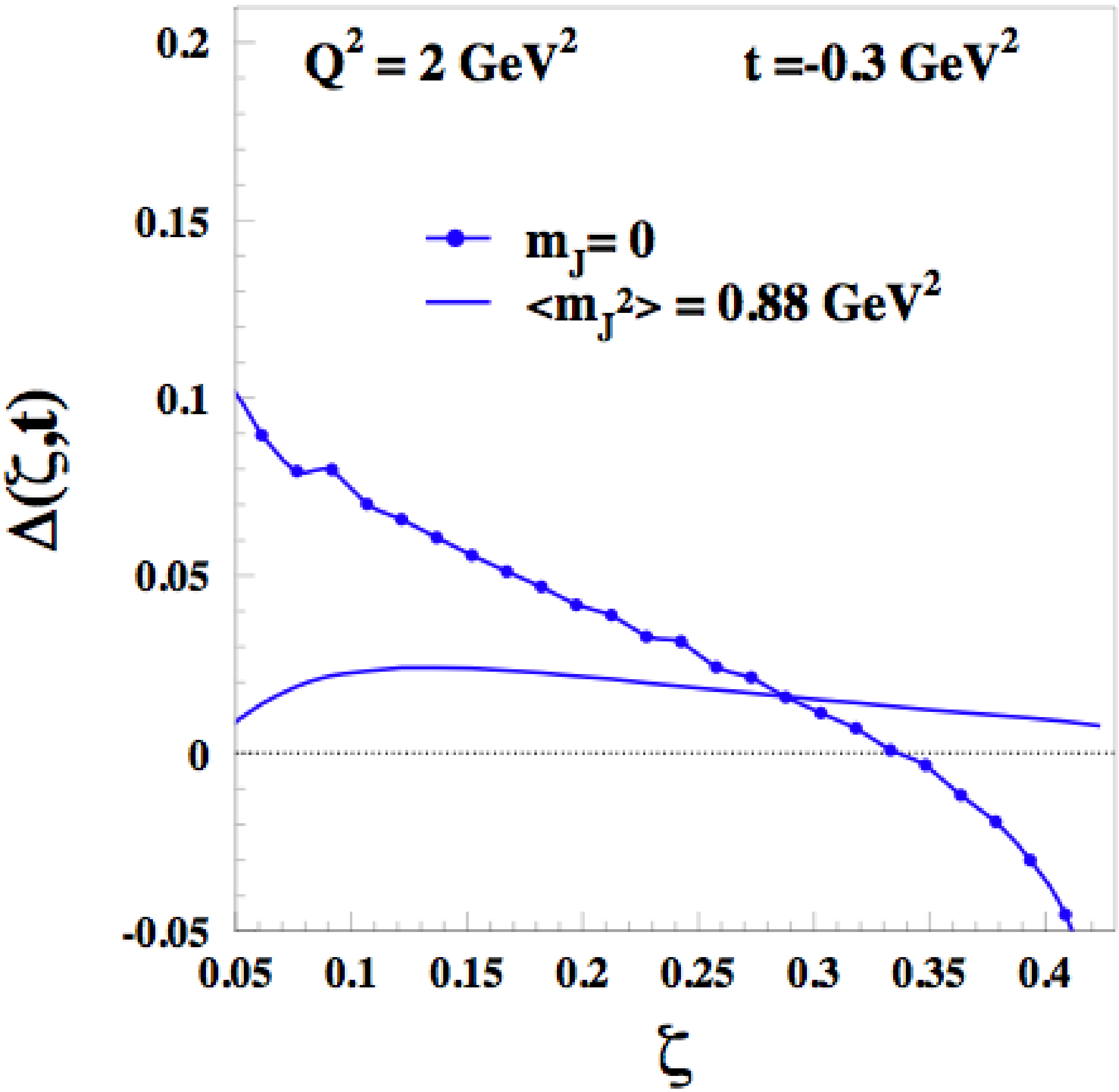


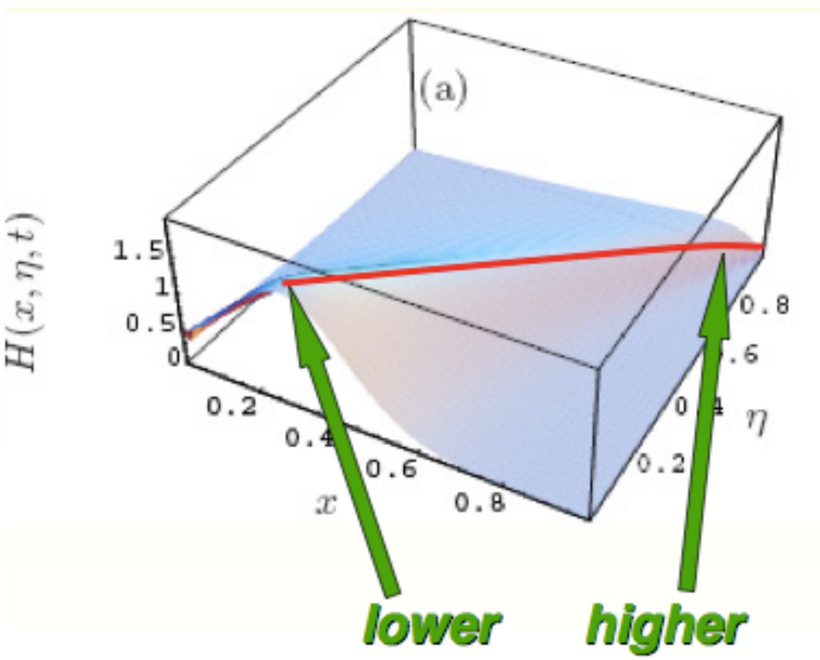
FIG. 2. The amplitude for $\gamma^* p$ scattering into two jets with fixed masses.



$Q^2 = 2\ GeV^2$
$t = -0.3\ GeV^2$
$m_J = 0$
$\langle m_J^2 \rangle = 0.88\ GeV^2$
$\Delta(\zeta, t)$
ζ
0.2
0.15
0.1
0.05
0
-0.05
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

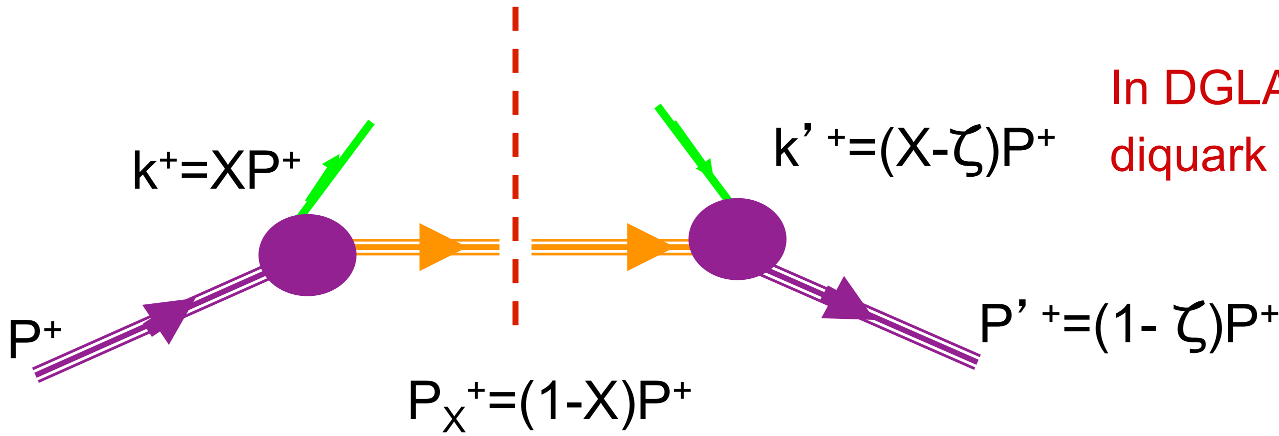
Summary of part 2: dispersion relations cannot be applied straighforwardly to DVCS.

The “ridge” does not seem to contain all the information

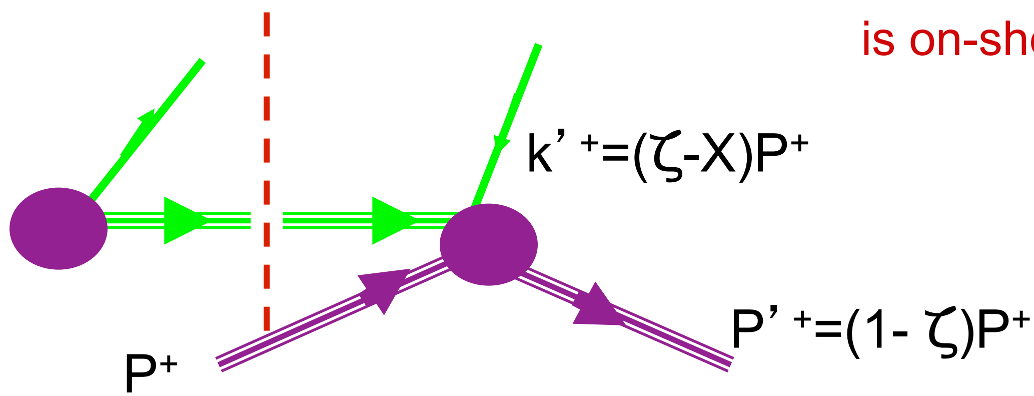


DVCS: Amplitude, no optical theorem

$$\mathcal{F}_q = \mathrm{Re}\,\mathcal{F}_q + i\,\mathrm{Im}\,\mathcal{F}_q$$



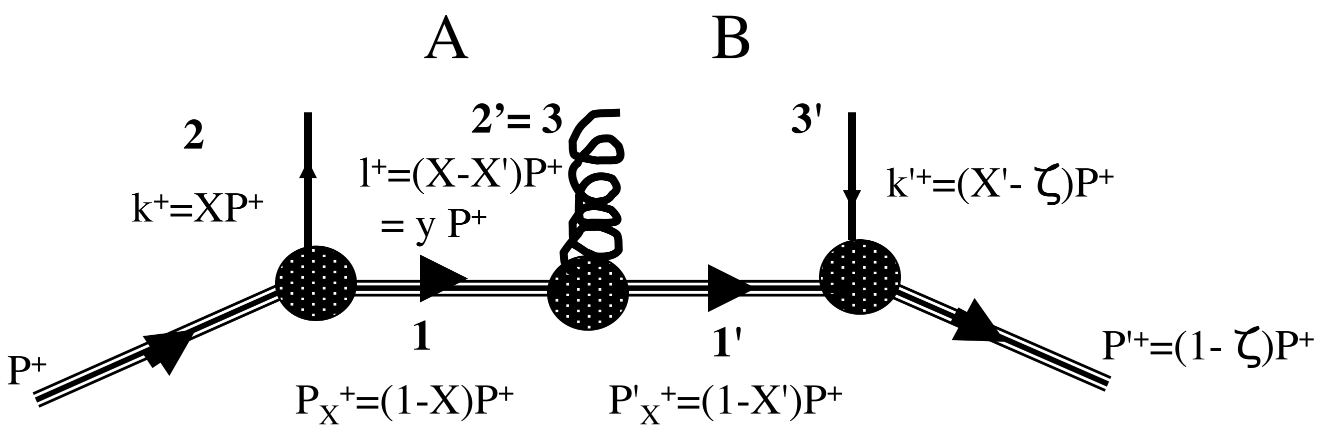
In DGLAP region spectator with diquark q. numbers is on-shell

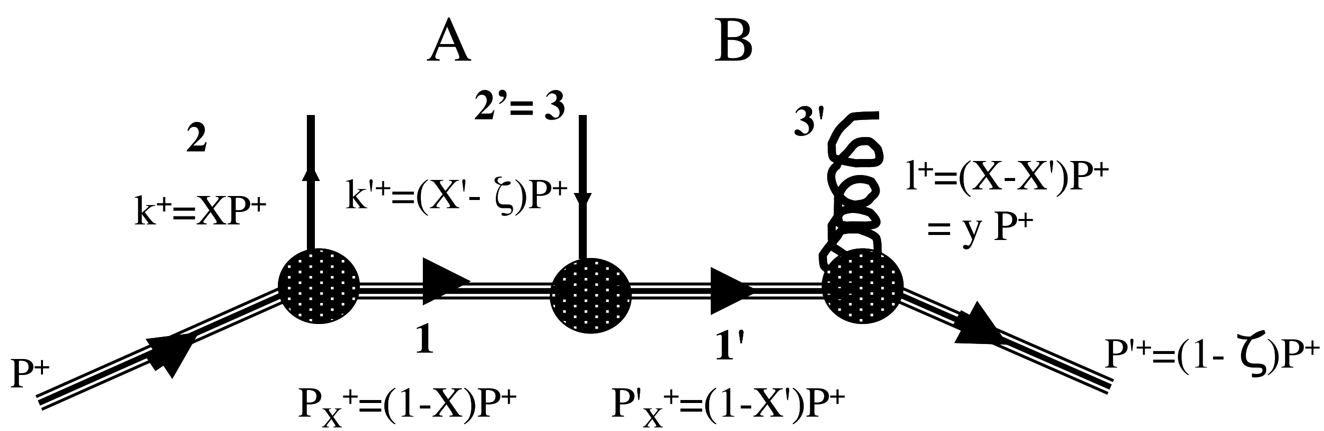


In ERBL region struck quark, k, is on-shell

Goldstein, Liuti arXiv 1006.0213: DIS/forward case discussed by Jaffe NPB(1983)
Diehl, Gousset:

In order to give a partonic interpretation one needs to introduce multiparton configurations ➔ FSI

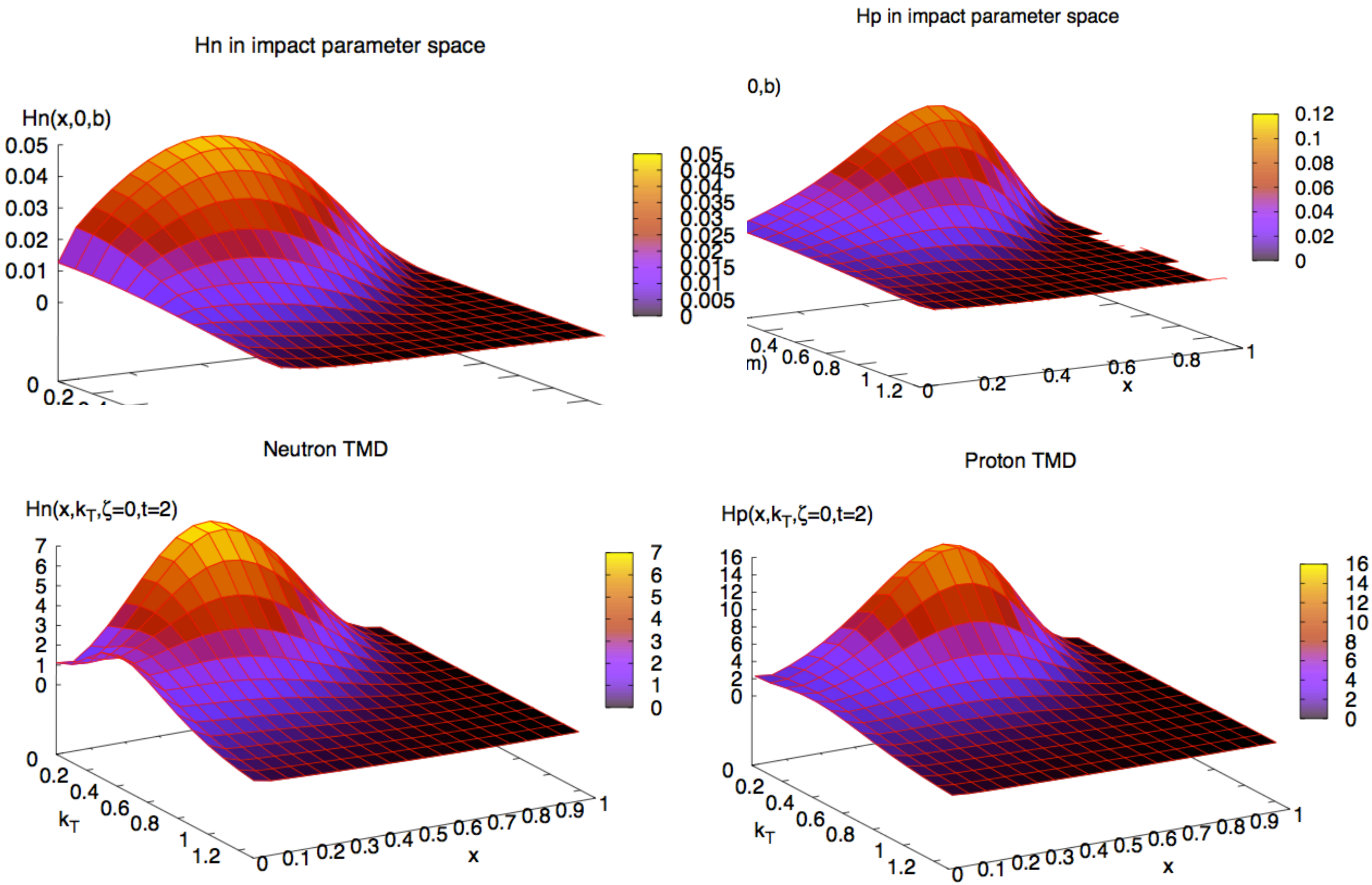




Practical Issues: Extraction of Wigner Dist'ns and OAM from Experiment

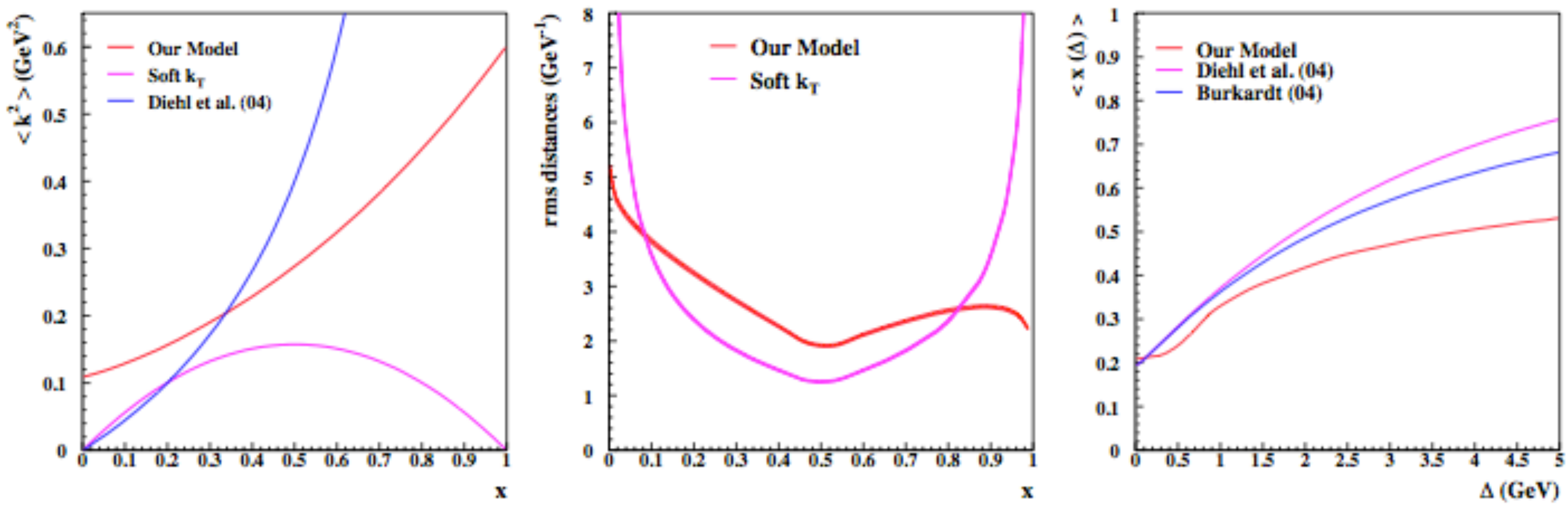
Flexible Parametrization

O. Gonzalez Hernandez using Goldstein et al. arXiv:1012.3776 (Gary's talk)



Slices of Wigner Distn's

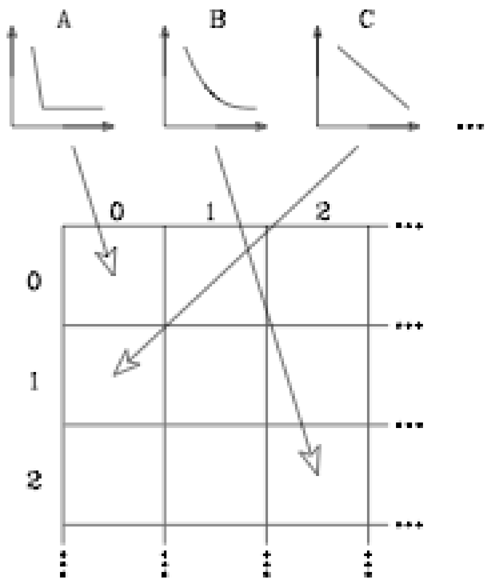
2 Simonetta Liuti: Study of Parton Interactions in Nuclei using Wigner Distributions

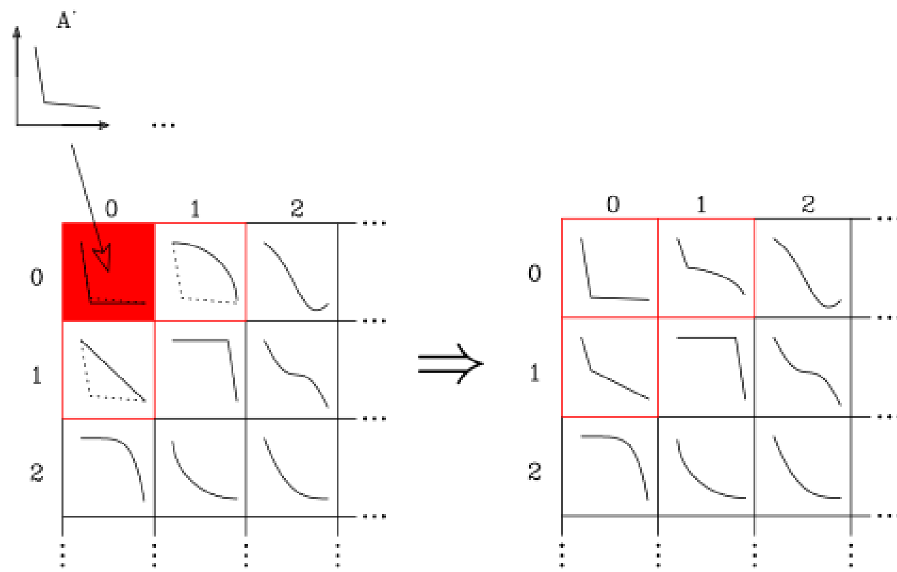


EIC Working Group, Editors: K. Hafidi et al.

Wigner Distn's: multiparticle systems that evolve from a large and varied number of initial conditions.

Multidimensional problem needs a carefully aimed Neural Network approach: Self-Organizing Maps





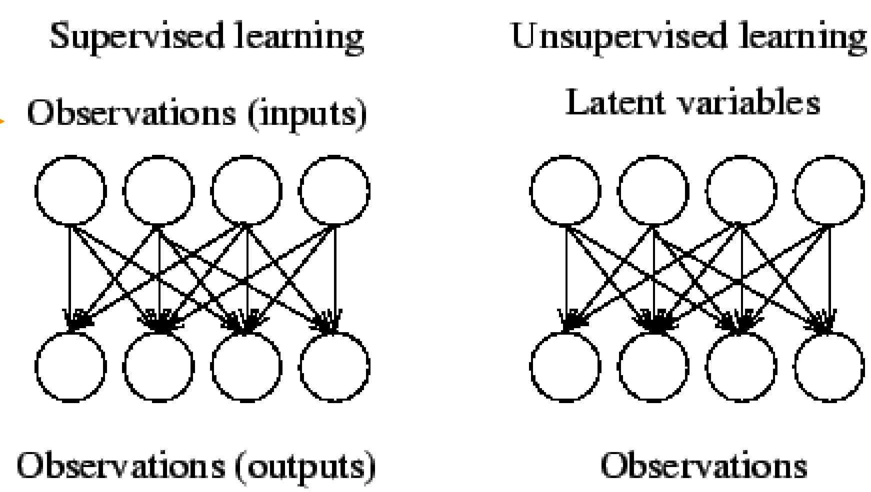
Initialization: functions are placed on map

Training: "winner" node is selected, Learning: adjacent nodes readjust according to similarity criterion

Supervised Learning → A set of examples is given.
The goal is to force the data
To match the examples as closely as possible.
The cost function includes information about the domain

Unsupervised Learning → No a priori examples are given.
The goal is to minimize the cost function by similarity relations, or by finding how the data cluster or self-organize
➔ global optimization problem

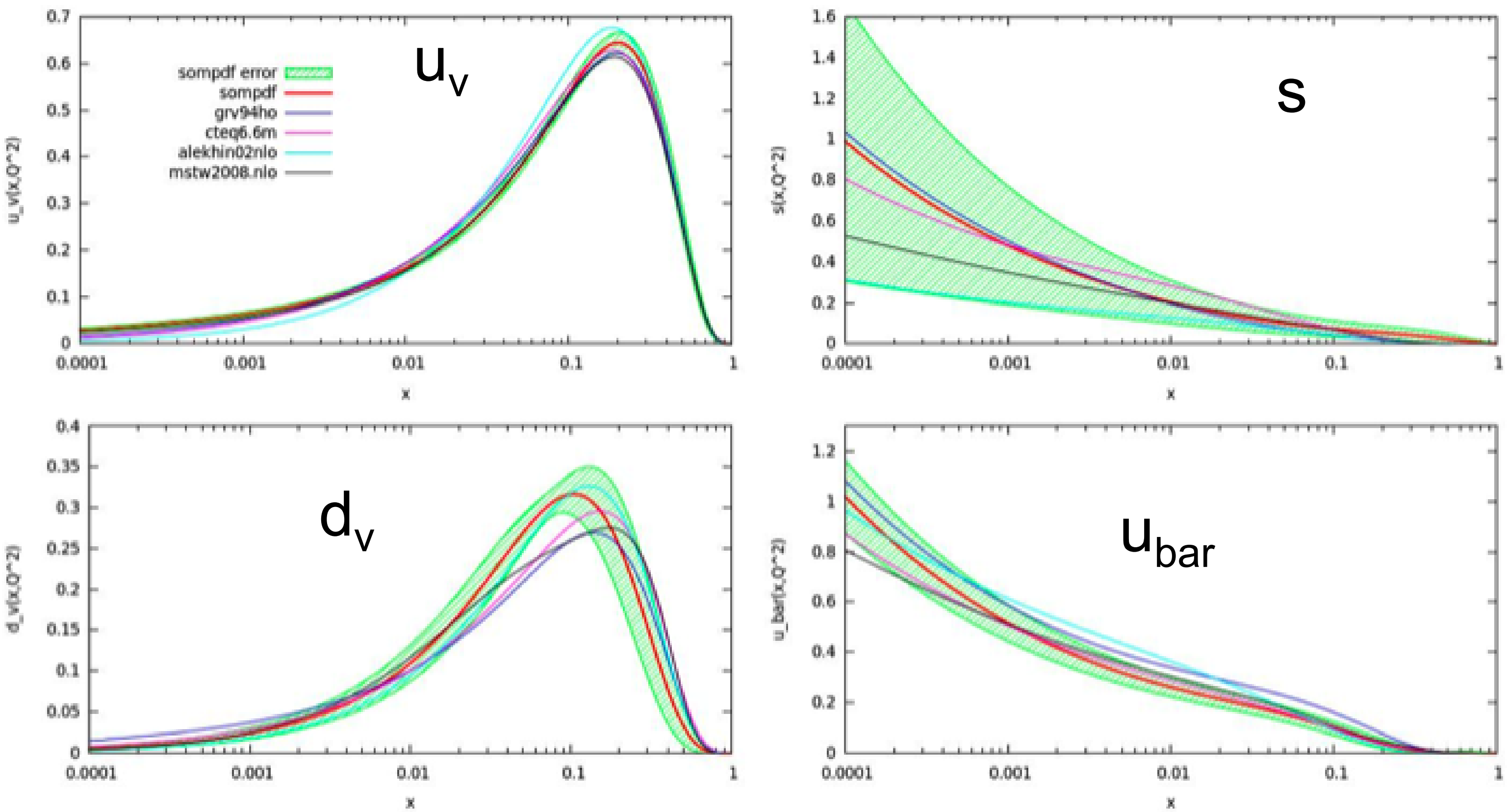
Important for PDF analysis!
If data are missing it is not possible to determine the output!



Preliminary Results (PDFs)

(D. Perry, DIS 2010 and MS Thesis 2010, K. Holcomb, Exclusive Processes Workshop, Jlab 2010)

$Q^2 = 7.5\ GeV^2$



OAM

Work in progress related to Taneja, Kathuria, S.L., Goldstein, arXiv:1101.0581

Ji $$\frac{1}{2} = L_z^q + \Delta\Sigma_q + J_g$$ $\longrightarrow$ GPDs

Jaffe Manohar $$\frac{1}{2} = \mathcal{L}_z^q + \Delta\Sigma_q + \Delta G + \mathcal{L}_z^g$$

$$\mathcal{L}_z^q \neq L_z^q$$

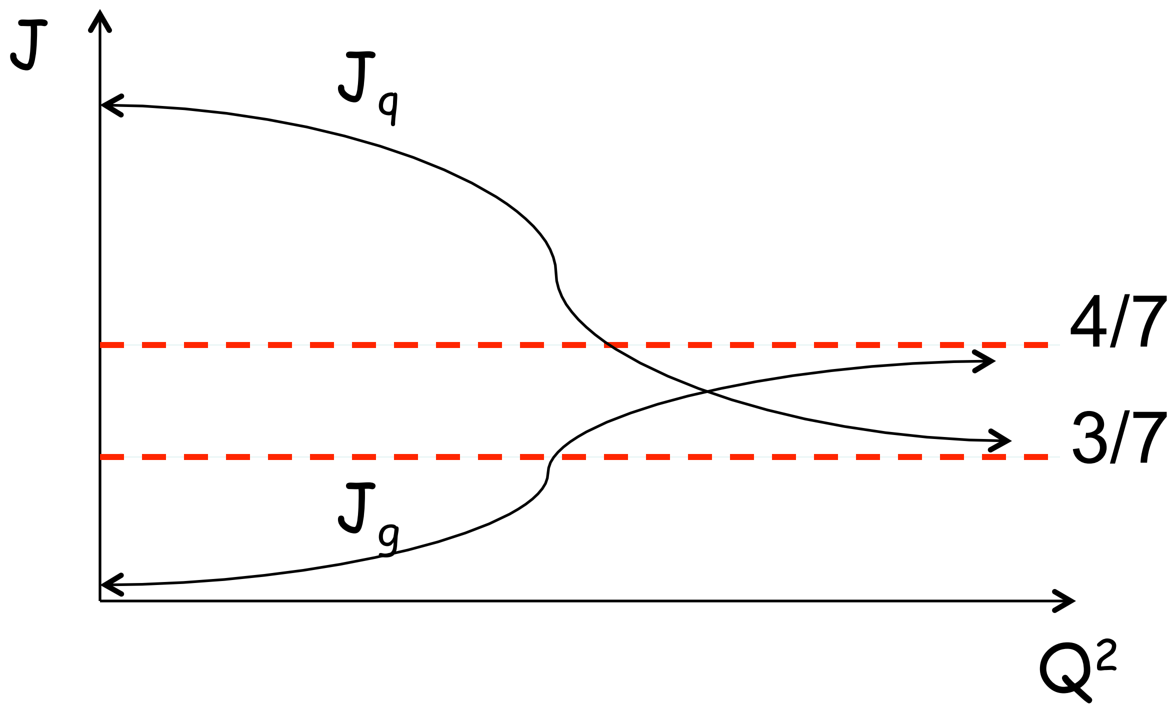
How big/important is the difference?
Burkardt and BC, 2009

No difference in scalar diquark models, difference appears including axial vector diquarks because a "vector potential" is present in this case
➔difference between covariant derivative and derivative

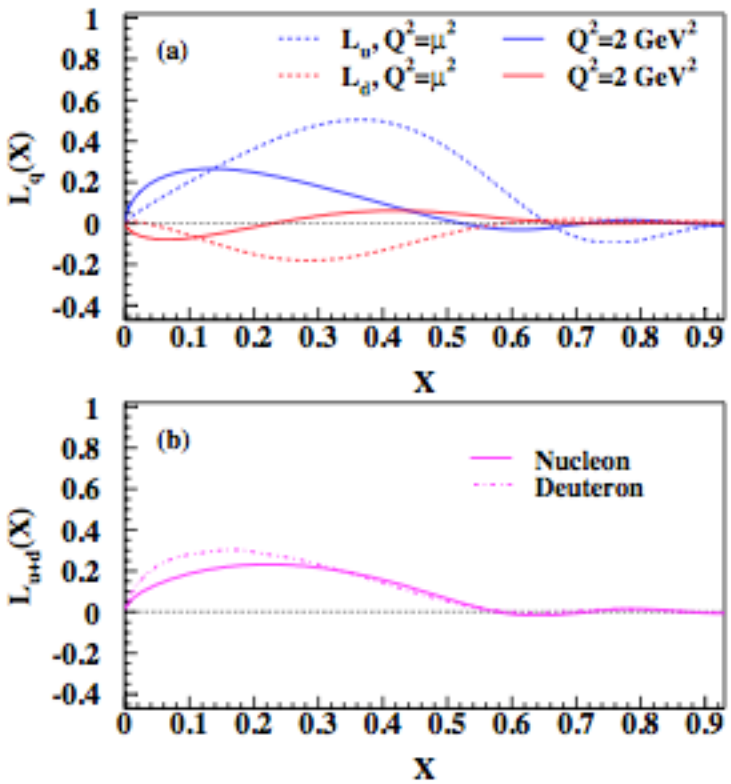
Our point of view:

One needs to consider PQCD evolution.

There exists a scale where no initial gluons are present. At Q_o^2 both models agree, disagreement will show up through evolution.



Will not comment on other sum rules, Chen et al, Wakamatsu



New sum rule for spin 1 system ➔deuteron

$$J_q = \frac{1}{2}\int dx\, x\,[H_q(x,0,0) + E_q(x,0,0)]\,, \longrightarrow J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

+ H.O.

$F_1+F_2= G_M$

G_M

Conclusions

✓ We uncovered a non-trivial partonic interpretation of GPDs
FSI important ➔ underlying connection with TMDs

✓Dispersion relations are not directly applicable: all information is not on the "ridge". Comprehensive measurement (real and imaginary parts) is important.

✓ Extraction of Wigner Distributions from data based on GPD flexible parametrization ➔ future, self-organizing maps (SOMGPDs)

✓ OAM with Jaffe-Manohar and Ji approaches: interesting relations from looking at spin 0, spin ½ and spin 1 (nuclei)